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Mean Cordial Labeling, Friendly labeling and Zero-edge magic labeling of Corona Graph C_noK_1

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Abstract: **Graph Theory is one of the branches of Mathematics. Graphs considered in Graph theory are discrete structures consisting of points and lines which connect these points. Graph labeling is assignment of numbers to the points and lines. Points are called vertices and lines are called edges. The graphs labeled using the vertices or edges of the graph is called labeled graph. Labeled graphs are used as models in different areas. One of the type of graph is Corona Graph C_noK_1 which has cycle C_n and n-copies of K_1 is attached to each vertex of C_n. In this paper, Mean Cordial labeling, Friendly labeling and Zero-edge magic labeling of Corona Graph will be discussed.**

Index Terms: **Corona Graph C_noK_1, Friendly labeling, Mean Cordial labeling, Zero-edge magic labeling.**

I. INTRODUCTION

In a graph G, an assignment of integers to the vertices or edges or both using certain conditions is called labeling of graph . Most of the terminologies and notations are taken from Harary [3]. Some survey of graph labelings is done from Gallian [2]. Let V(G) denote the vertex set and E(G) denote the set of edges. The graph G is denoted as ordered pair of V(G) and E(G) i.e. G = (V(G), E(G)). There are many types of labelings techniques. Some of them are E-cordial labeling, prime labeling, harmonic labeling, magic labeling etc. applied to certain classes of graphs. Some of them are vertex labelings and edge labelings are induced from it while some are edge labelings and vertex labelings are induced from it. Corona graphs **C_noK_1** are simple and undirected graphs. In this paper, mean cordial labeling, friendly labeling and zero-edge magic labeling of Corona graphs **C_noK_1** are discussed. The labelings are vertex labelings and edge labelings are induced from it. These labelings can be given to many classes of graph.

II. PRELIMINARIES AND NOTATION

Here, in this part, the basic definitions are given for development of paper.

Definition 2.1 [3]**: Cycle graph C_n**: A connected graph with n edges in which initial and final verex are same.

Definition 2.2: Corona graph C_noK_1 [3]**:** The corona $G_1{}^oG_2$ of two graphs G_1 and G_2 is a graph G obtained by taking one copy of G_1 which has p_1-vertices and p_1-copies of G_2 and then joining i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2.

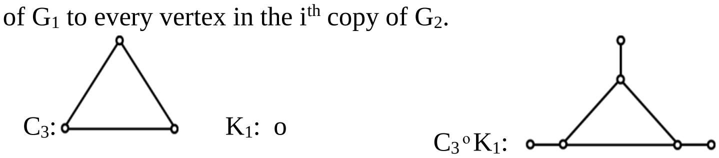


Figure 1: Corona $C_3{}^oK_1$

III. MAIN RESULTS

3.1. Mean Cordial Labeling of corona graph $C_n{}^oK_1$.

Definition 3.1.1: Mean Cordial Labeling [7]**:** For each edge uv of graph G assign the label $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ (ceiling function). Then the map f : V(G)→{0,1, 2}is called mean cordial labeling if $\left|v_f(i) - v_f(j)\right| \leq 1$ and $\left|e_{f*}(i) - e_{f*}(j)\right| \leq 1$; i, j ∈ {0,1, 2} where $v_f(x)$ and $e_{f*}(x)$ denote the number of vertices and edges respectively labeled with x .

Definition 3.1.2: Mean Cordial Graph [7]**:** A graph G which admits mean cordial labeling is called mean cordial graph.

Theorem 3.1.3 : The corona graph **C_noK_1** is mean cordial graph for n ≡ 2 (mod 3) and n ≥ 3.

Proof : Let $\{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$ be the set of vertices of G = $\mathbf{C_n o K_1}$.

The vertices on cycle C_n are $u_1, u_2, \ldots, u_n$ while $v_1, v_2, \ldots, v_n$ are pendant vertices adjacent to $u_1, u_2, \ldots, u_n$ respectively.

Define labeling function $f : V(G) \rightarrow \{0,1,2\}$ of vertices of G = $\mathbf{C_n o K_1}$ as follows.

$f(u_i) = 0$ for $1 \le i \le \frac{n+1}{3}$ [cycle vertices]

$= 2$ for $\frac{n+4}{3} \le i \le \frac{2n+2}{3}$

$= 1$ for $\frac{2n+5}{3} \le i \le n$.

$f(v_i) = 0$ for $1 \le i \le \frac{n+1}{3}$ [pendant vertices]

$= 2$ for $\frac{n+4}{3} \le i \le \frac{2n-1}{3}$

$= 1$ for $\frac{2n+2}{3} \le i \le n$.

This labeling of vertices induces the edge labelings as follows:

$f(e_i) = f(u_i u_{i+1}) = 0$ for $1 \le i \le \frac{n-2}{3}$ [cycle edges]

$= 2$ for $\frac{n+4}{3} \le i \le \frac{2n+2}{3}$

$= 1$ for $\frac{2n+5}{3} \le i \le n-1$.

$f(e_n) = f(u_n u_1) = 1$ and $f(e_i) = 1$ when $i = \frac{n+1}{3}$

$f(e_i') = f(u_i v_i) = 0$ for $1 \le i \le \frac{n+1}{3}$ [pendant edges]

$= 2$ for $\frac{n+4}{3} \le i \le \frac{2n+2}{3}$

$= 1$ for $\frac{2n+5}{3} \le i \le n$.

This vertex labeling and induced edge labels shows that

$v_f(0) = \frac{2n+2}{3}$, $v_f(1) = \frac{2n-1}{3}$,

$v_f(2) = \frac{2n-1}{3}$ and

$e_{f^*}(0) = \frac{2n-1}{3}$, $e_{f^*}(1) = \frac{2n-1}{3}$,

$e_{f^*}(2) = \frac{2n+2}{3}$

which satisfies $|v_f(i) - v_f(j)| \le 1$ and $|e_{f*}(i) - e_{f*}(j)| \le 1$ for all i, j ∈ {0,1, 2}.

Hence, corona graph $\mathbf{C_n o K_1}$ is mean cordial graph for n ≡ 2 (mod 3) and n ≥ 3.

Remark 3.1.4: When n ≡ 0 (mod 3) and n ≡ 1 (mod 3) the graph cannot be mean cordial graph.

Case(i) When n ≡ 0 (mod 3), then $|V(C_n o K_1)| = 2n = |E(C_n o K_1)|$.

Also, $v_f(i) = \frac{2n}{3}$, for all i, ∈ {0,1, 2} while $e_{f^*}(0) = \frac{2n-3}{3}$, $e_{f^*}(1) = \frac{2n}{3}$,

$e_{f^*}(2) = \frac{2n+3}{3}$.

Hence, $|e_{f*}(i) - e_{f*}(j)| \le 1$ is not satisfied for all i,j∈{0,1, 2}.

Case(ii) When n ≡ 1 (mod 3), then $|V(C_n o K_1)| = 2n = |E(C_n o K_1)|$.

Also, $v_f(0) = \frac{2n-2}{3}$, $v_f(1) = \frac{2n+1}{3}$, $v_f(2) = \frac{2n+1}{3}$

while $e_{f^*}(0) = \frac{2n-5}{3}$, $e_{f^*}(1) = \frac{2n+1}{3}$, $e_{f^*}(2) = \frac{2n+4}{3}$.

Hence, $|e_{f*}(i) - e_{f*}(j)| \le 1$ is not satisfied for all i, j ∈ {0,1,2}.

3.2. Friendly labeling of corona graph $\mathbf{C_n \circ K_1}$.

Notations and terminologies in 3.2 are taken from[1].

Definition 3.2.1 :- Friendly labeling.
Let G be a graph and f: V(G) → Z_2 be a binary vertex labeling of G. For i ∈ Z_2 let, $v_f(i) = |f^{-1}(i)|$. Then the labeling f is said to be friendly if $|v_f(1) - v_f(0)| \le 1$.
$v_f(1)$ denotes number of vertices labeled 1.

Definition 3.2.2 :- Product – cordial index or pc-index.
Any friendly labeling f: V(G) → Z_2 induces an edge labeling f* : E (G) → Z_2 defined by f* (xy) = f(x) f(y) ∀ xy∈ E(G) .
For i ∈ Z_2, let $e_f*(i) = |f^{*-1}(i)|$ be the number of edges of G that are labeled i. The number pc(f) = $|e_{f^*}(1) - e_{f^*}(0)|$ is called the product-cordial index or pc-index of f.

Definition 3.2.3 :- Product-Cordial set or pc-set.
The product cordial set or pc-set of the graph G, denoted by PC(G), is defined by PC(G) = {pc(f) : f is friendly vertex labeling of G}.

Definition 3.2.4 :- Product-cordial graph.
A graph with friendly labeling is called product cordial graph.

Definition 3.2.5 :- Fully product-cordial or fully pc graph.
A graph G of size q is said to be fully product cordial or fully pc if PC(G) = $\{q - 2k : 0 \le k \le \lfloor q/2 \rfloor\}$ where $\lfloor x \rfloor$ denotes greatest integer less than or equal to x.

Example 3.2.6 : Friendly labeling of a graph.

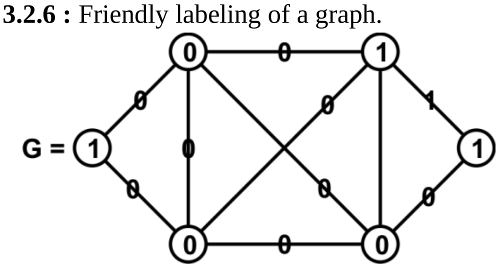


Figure 2 : Friendly labeling of graph G

Consider the graph G of figure 2 which has six vertices. The condition

$|v_f(1) - v_f(0)| \le 1$ implies that three vertices are labeled 0 and the other three 1. Product cordial index for this labeling is pc(f) = $|e_f(1) - e_f(0)| = |9-1| = 8$.

Therefore, PC(G)= { pc(f) : f is friendly vertex labeling of G}={4,6,8}.

This friendly labeling does not give fully product cordial graph.

Theorem 3.2.7 : For any $n \geq 3$, the corona graph $C_n \circ K_1$ have friendly-labeling and is fully product cordial. Also, $PC(C_n \circ K_1) = \{2(n-k): 0 \leq k \leq n\}$

Proof : Consider corona graph $C_n \circ K_1$ with vertex set V and edge set E. $|V| = 2n = |E|$

Let $\{u_1, u_2, \ldots, u_n\}$ be the vertices on the cycle C_n. Let $\{v_1, v_2, \ldots, v_n\}$ be the vertices pendant to $u_1, u_2, \ldots, u_n$ respectively. The labeling of corona graph can be done as follows:

1] For pc(f) = 0, label all vertices on cycle as 1 and all pendant vertices as zero. Here $v_f(0) = n = v_f(1)$ and $e_{f^*}(0) = n = e_{f^*}(1)$

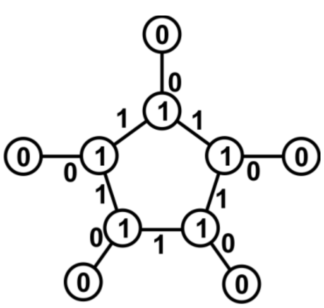
2] For pc(f) = 2n,label all vertices on cycle as zero and all pendant vertices as 1. Here $v_f(0) = n = v_f(1)$ and $e_{f^*}(0) = 2n$, $e_{f^*}(1) = 0$

3] For pc(f) = 2, 4, 6,8,......, 2n–2 and for i =1, 2,....., n–1

Label 'n-i' consecutive vertices on cycle by 1 and remaining 'i' vertices by zero. Label one pendant vertex incident to 1 by 1 and all the remaining vertices pendant to label 1 will be labeled 0. Label 'i-1' pendant vertices incident to vertex with label zero by 1 and remaining all pendant vertices incident to vertex with label zero by 0. Here for $1 \leq i \leq n-1$, $v_f(0) = n = v_f(1)$, $e_{f^*}(0) = n + i$ and $e_f^*(1) = n - i$. Therefore $pc(f) = 2i$. The above **labeling makes $C_n \circ K_i$ fully product-cordial.**

Illustration 3.2.8 :- Consider $C_5 \circ K_1$.

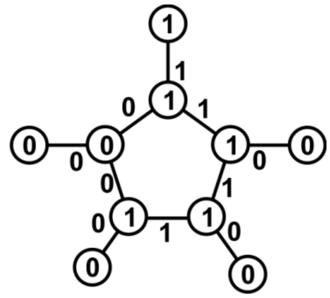
Here; n = 5.



1) $v_f(0) = v_f(1) = 5$

$e_{f^*}(0) = 5 = e_{f^*}(1)$

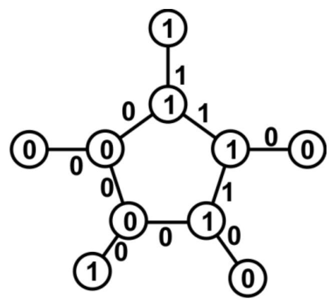
$\therefore pc(f) = 0$.



2) $v_f(0) = v_f(1) = 5$

$e_{f^*}(1) = 4$, $e_{f^*}(0) = 6$

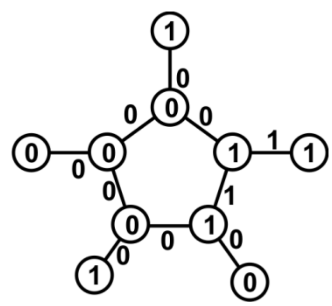
$\therefore pc(f) = 2$.



3) $v_f(0) = 5 = v_f(1)$

$e_{f^*}(1) = 3$, $e_{f^*}(0) = 7$

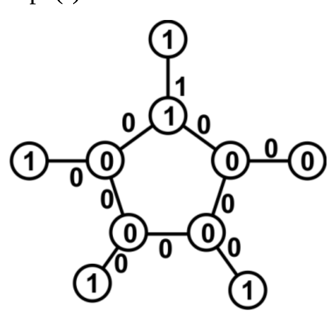
$\therefore pc(f) = 4$.



4) $v_f(0) = v_f(1) = 5$

$e_{f^*}(1) = 2$, $e_{f^*}(0) = 8$

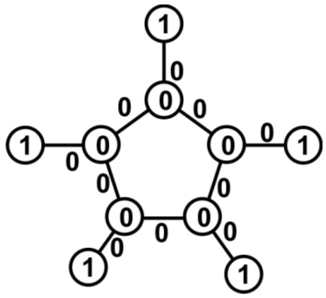
$\therefore pc(f) = 6$.



5) $v_f(0) = v_f(1) = 5$

$e_{f^*}(1) = 1$, $e_{f^*}(0) = 9$

$\therefore pc(f) = 8$.



6) $v_f(0) = v_f(1) = 5$

$e_{f^*}(0) = e_{f^*}(1) = 0$

$\therefore pc(f) = 10$.

Thus, $PC(C_5{}^o K_1) = \{pc(f) : f$ is friendly labeling of $C_n{}^o K_1\} = \{0,2,4,6,8,10\}$

$= \{q - 2k : 0 \le k \le \lfloor q/2 \rfloor\}$ where

$q = 10$.

Hence, $C_5{}^o K_1$ is fully product cordial graph.

3.3. Zero-edge magic labeling of corona graph $C_n{}^o K_1$.

Notations and terminologies in 3.3 are taken from[4].

Definition 3.3.1: Zero-edge magic labeling.

Let G=(V, E) be a graph. Let $f : V \to \{-1, 1\}$ and $f^* : E \to \{0\}$ such that all $uv \in E$, $f^*(uv)=f(u) + f(v) = 0$ then the labeling is said to be zero–edge magic labeling of graph G.

Definition 3.3.2: Zero-edge magic graph.

A graph which admits zero edge magic labeling is called zero-edge magic graph.

Theorem 3.3.3: The corona graph $C_n{}^o K_1$ is zero edge magic for n even.

Proof : Let $u_1, u_2, \ldots, u_n$ be vertices of C_n and $v_1, v_2, \ldots, v_n$ be the vertices pendant to $u_1, u_2, \ldots u_n$ respectively. Define labeling $f : V \to \{-1, 1\}$, where $V= \{u_1, u_2, \ldots u_n\}$ as follows : $f(u_i) = (-1)^i \ \forall\ 1 \le i \le n$ and $f(v_i) = (-1)^{i+1} \ \forall\ 1 \le i \le n$

Then, $f^*(u_i u_{i+1}) = f(u_i) + f(u_{i+1})$

$= (-1)^i + (-1)^{i+1}$

$= -1 + 1 = 0$

if i is odd , $1 \le i \le n-1$

$= 1 + (-1) = 0$

if i is even, $1 \le i \le n-1$

$f(u_n u_1) = 0$.

And $f^*(u_i v_i) = f(u_i) + f(v_i)$

$= (-1)^i + (-1)^{i+1}$

$= -1 + 1 = 0$

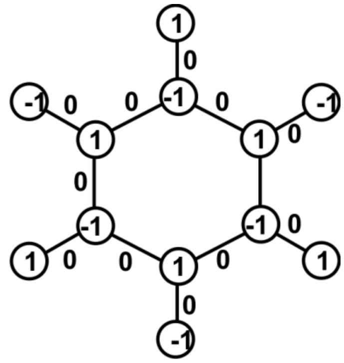
if i is odd

$= 1 + (-1) = 0$

if i is even. **Hence, $C_n{}^o K_1$ admits zero edge magic labeling and $C_n{}^o K_1$ is zero edge magic graph.**

Illustration 3.3.4: Consider $C_6{}^o K_1$.

Here n = 6.



$f(u_i) = (-1)^i$; $1 \le i \le 6$

$f(v_i) = (-1)^{i+1}$; $1 \le i \le 6$. Then

$\therefore C_6{}^o K_1$ is zero-edge magic graph.

Remark 3.3.5: When n is odd, one of the induced edge label is either 2 or -2, hence the graph is not zero–edge magic graph.

CONCLUSION AND FUTURE RESEARCH DIRECTIONS

A particular class of graph admit different labelings techniques. Similar results can be obtained in the context of different graphs. Algorithms can be developed for these labeling. Applications of these labelings with covering problem to project Management.

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